How plural is the plural economy of Bolivia? Constructing a plural economy indicator with fuzzy sets

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Abstract

An indicator that measures the compliance with the constitutional principles of a plural economy is proposed. An inference system based on fuzzy sets was used to calculate the indicator. The fuzzy system summarizes the principles of income redistribution and environmental sustainability into an overall measure of plural economy that allows to objectively judge the change towards a plural economy in Bolivia.

¿Cuán plural es la economía plural de Bolivia? Construyendo un indicador de economía plural con conjuntos difusos

Resumen

Se propone un indicador que mide el cumplimiento de los principios constitucionales de una economía plural. Para calcular el indicador se usó un sistema de inferencia basado en conjuntos difusos. El sistema difuso resume los principios de la redistribución del ingreso y la sostenibilidad ambiental en una medida global de economía plural que permite juzgar objetivamente el cambio hacia una economía plural en Bolivia.

Clasificación JEL: *C02, P40*
Palabras clave: *Indicador de economía plural, conjuntos difusos*

I. Introduction

As part of a series of changes of the economic model of Bolivia, the government of this country enacted in 2009 a new State Constitution with the principles of a plural economy. According to this new Constitution, the plural economy of Bolivia comprises different forms of economic organization and it is based on the principles of *complementariness, reciprocity, solidarity, redistribution, equity, legal certainty, sustainability, equilibrium, justice* and *transparency.*[1]

Antagonistic political parties take different positions about the change to a plural economy in Bolivia. Opposite parties untiringly argue about the existence of this change, and they eventually link their discussion to the principles of the Bolivian plural economy, because if these principles were not met, then it would not be easy to claim a shift towards a more pluralistic economy. Being the principles of a plural economy (henceforth, PPE) vague concepts, any verbal discussion about the compliance with the principles would be subjective and ultimately unproductive. A more scientific approach is to use mathematical tools to objectively measure the degree of compliance with these principles. In this sense, this paper proposes a mathematical indicator of the plurality of an economy based on fuzzy sets. This indicator can be used to objectively measure the degree of compliance with the PPE, and thus assert the existence of a change to a plural economy in Bolivia.

Since the purpose of this paper is to make a methodological contribution, and in order to keep a simple exposition of the techniques, only two of the ten principles were modeled. A complete analysis of the ten PPE would require a full elicitation of 30 fuzzy membership functions and the plural economy indicator would be a 11-polytope (defined in a 11-dimensional hypercube). Due to this, visualizing the plural economy indicator would not be as straightforward as in the Figure 4 of this paper.

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1   New Constitution of the State, Part Four (Economic Structure and Organization of the State), Title I (Economic Organization of the State), Chapter One (General Dispositions), Article 306.

Section II offers some background on the issues behind the concept of a plural economy and their relationship with fuzzy logic. The methods to construct the plural economy indicator are described in section III. Section IV contains the results of the indicator, and Section V concludes.

MATLAB codes which compute the plural economy indicator and replicate the empirical work reported in this paper are available upon request.

II.   Background

A theoretical approach to the understanding of a 'plural economy' appears to precede the more subtle linguistic characterization of a plural economy contained in the Bolivian new State Constitution of 2009. The Bolivian concept of a 'plural economy' seems to have its roots in the economic theory of *welfare pluralism* or *mixed economy of welfare* that highlights the role of the nonprofit sector in welfare and claims that a balanced social economy must be beyond the so-called 'market fundamentalism of neoclassical economics'. According to this approach, welfare is provided by different societal actors, e.g. the government, private sources or social cooperatives. See, *inter alia,* Evers and Laville (2004), Heitzmann (2006), Stiglitz (2009), Bresser-Pereira (2010) or Etxezarreta and Bakaikoa (2011).

In Bolivia, the implementation of a plural economy starts in 2006, when a left-wing government changed the Bolivian economic structure in a way that sought to oppose to the so-called neoliberal economic structure. In this alternative model, the economy is comprised by the interaction of social, communitary, private, and State economic structures (Pardo, 2009), and the State is conceptualized as the leader in the strategic sectors that deal with the generation and distribution of wealth (García, 2011). Nevertheless, the concept of 'plural economy' emerges officially from the new State Constitution of Bolivia enacted in 2009. According to this new Constitution, a plural economy is based on the principles of *complementariness, reciprocity, solidarity, redistribution, equity, legal certainty, sustainability, equilibrium, justice,* and *transparency*. Evidently, these principles are linguistic concepts that contain ambiguities. Thus,

policy reasoning with such imprecise concepts may not be clear and obvious, but rather fuzzy.

Fuzzy logic can be defined as a precise logic of imprecision and an approximate reasoning, with high power of *precisiation*[2], of what is semantically imprecise (Zadeh, 2008).[3] Fuzzy sets, being mathematical tools to handle linguistic variables, are ideal for constructing objective indicators to condense polymorphous concepts. This is why fuzzy logic is often used to construct socioeconomic indicators that measure, *inter alia*, bankruptcy, well-being, contamination risk, sustainability, market bidding adjustments or stock market fluctuations. Östermark (1999) for example, developed an inference system for bankrupt firms, based on fuzzy multigroup classification. Chiappero (2000) tried to make some progress towards the possibility of realizing a multidimensional assessment of Amartya Sen's concept of well-being with the use of fuzzy sets theory. In Castignani et al. (2004), pesticide contamination risk was calculated with a fuzzy logic indicator and the economic costs were assessed by gross margin differences among farm typologies. Phillis and Andriantiatsaholiniaina (2001) used fuzzy logic to construct an inference system that takes ecological (land, water, air, and biodiversity) and human (economical, social, educational, and political) inputs, and then combined these with the aid of fuzzy logic to provide an overall measure of the degree of sustainability of the system under examination (see also Andriantiatsaholiniaina et al., 2004). He et al. (2006) employed heuristic fuzzy rules to emulate the reasoning of artificial agents that dynamically adjust its bidding behavior to effectively respond to changes in the supply and demand of a marketplace. Finally, with a self-organized fuzzy neural network, Bollen et al. (2011) used the twitter mood to predict the stock market, using the Dow Jones Industrial

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2   Zadeh (2012) defines *precisiation* as the construction of computational/mathematical models of words, phrases, propositions, questions and other types of semantic entities.

3   The history of fuzzy logic can be traced back to the Bertrand Rusell's opinion on the Cretans paradox. In the year 1920, Łukasiewicz worked out a multivalued logic in which statements can take on fractional values between the 'ones and zeros' of binary logic. In 1973, quantum philosopher Max Black drew the first fuzzy set curves, and almost 30 years later Zadeh worked out a complete algebra for fuzzy sets on a paper called "fuzzy sets", term that gave name to this field. Fuzzy logic was popularized by the use of fuzzy sets in Japan to control systems. In 1988, Hitachi turned over control of a subway in Sendai, Japan, to a fuzzy system. In 1992, the Ministry of International Trade and Industry estimated that Japan produced about $2 billion worth of fuzzy products (Kosko and Isaka, 1993).

Average index.[4] In this study, the PPE are used as inputs of the fuzzy system, thus offering the possibility of obtaining an overall (numerical) indicator that summarizes the degree of compliance with the welfare pluralism of the principles of a plural economy.

III.   Methods [5]

Let x be an economic indicator that measures a PPE. This and other n-indicators x_1, ... , x_n can be summarized into a single plural economy indicator with an inference system based on fuzzy sets. In this system, a $1 \times n$ vector ξ with values of the n-economic indicators is placed into input membership functions. Based on a set of rules, these input functions map the input indicators to an output membership function. The output function translates through fuzzy sets the verbal degree of compliance with the PPE to a numerical indicator bounded between 0 and 1.

III.1. Fuzzy sets

Let A denote a set, x the element of any set and U the universal set of all objects under consideration. To define a fuzzy set is necessary to define the *characteristic function* of a set.

Definition 1 *Characteristic function* *(membership function). The characteristic function $\mu_A(x)$ of a set $A \subset U$ is a membership rule that characterizes the elements (members) of a set , taking only two values 0 and 1, indicating whether or not $x \in U$ is a member of A:*

$$\mu_A(x) = \begin{cases} 1 & for\ x \in A \\ 0 & for\ x \notin A \end{cases} \qquad (1)$$

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4   In Bolivia, the use of fuzzy logic is almost nil; the only reference that the author found is Avilés (2009).

5   This section is based on Bojadziev and Bojadziev (2007), Sivanandam et al. (2007), and Guney and Sarikaya (2009).

As $\mu_A(x) \in \{0,1\}$, the characteristic function only indicates if an element belongs or not belongs to A. Fuzzy sets unstrain this crisp membership rule.

Definition 2 *Fuzzy sets. Let A be a classical set of the universe U. A fuzzy set $\mathcal{A}$ is defined by a set of ordered pairs (a binary relation),*

$$\mathcal{A} = \{(x, \mu_A(x)) \mid x \in \mathcal{A}, \mu_A(x) \in [0,1]\} \qquad (2)$$

where $\mu_A(x)$ is a membership function that specifies the degree to which any element x in A belongs to the fuzzy set $\mathcal{A}$, and it ties each element x in A to a real number $\mathbb{R}$ in the interval $[0,1]$.

Fuzzy sets allow to measure numerically the *degree* of membership to the set A, because in contrast to the function $\mu_A(x) \in \{0,1\}$, the membership function of a fuzzy set $\mu_A(x)$ is continuous between 0 and 1.

III.2. *Membership functions*

A spline-based z-function, a s-function and a gaussian function can be used as input membership functions. The parameters of the spline-based z-function $z\,(x;\zeta_1, \zeta_2)$,

$$z\,(x;\zeta_1,\zeta_2) = \begin{cases} 1, & x \leq \zeta_1 \\ 1 - 2\left(\frac{x-\zeta_1}{\zeta_2-\zeta_1}\right)^2, & \zeta_1 \leq x \leq \frac{\zeta_1+\zeta_2}{2} \\ 2\left(\frac{x-\zeta_1}{\zeta_2-\zeta_1}\right)^2, & \frac{\zeta_1+\zeta_2}{2} \leq x \leq \zeta_2 \\ 0, & x \geq \zeta_2 \end{cases} \qquad (3)$$

and the spline-based s-function $s\,(x;\omega_1, \omega_2)$,

$$s\,(x;\omega_1,\omega_2) = \begin{cases} 1, & x \leq \omega_1 \\ 2\left(\frac{x-\omega_1}{\omega_2-\omega_1}\right)^2, & \omega_1 \leq x \leq \frac{\omega_1+\omega_2}{2} \\ 1 - 2\left(\frac{x-\omega_1}{\omega_2-\omega_1}\right)^2, & \frac{\omega_1+\omega_2}{2} \leq x \leq \omega_2 \\ 0, & x \geq \omega_2 \end{cases} \qquad (4)$$

can be calibrated to model the bounds of the economic indicators. The parameters σ, ψ of the Gaussian function $g\,(x;\,\sigma,\,\psi)$

$$g\left(x;\sigma,\psi\right)=e^{\frac{-(x-\psi)^2}{2\sigma^2}} \tag{5}$$

can be estimated with historical data of economic indicators, in order to measure the average of the indicators and the spread around this average.

For the output of the inference system, a trapezoidal function $trap\,(x;\,\delta_1,\,\delta_2,\,\delta_3,\,\delta_4)$

$$trap\left(x;\delta_1,\delta_2,\delta_3,\delta_4\right)=\max\left[\min\left(\frac{x-\delta_1}{\delta_2-\delta_1},1,\frac{\delta_4-x}{\delta_4-\delta_3}\right),0\right] \tag{6}$$

models the non-compliance and the full compliance with the PPE, and a triangular function $tri\,(x;\,\tau_1,\,\tau_2,\,\tau_3)$,

$$tri\left(x;\tau_1,\tau_2,\tau_3\right)=\max\left[\min\left(\frac{x-\tau_1}{\tau_2-\tau_1},\frac{\delta_3-x}{\delta_3-\delta_2}\right),0\right] \tag{7}$$

models the middle compliance. Intuitively, these membership functions allow to translate linguistic vague concepts as "high income inequality" to objective, continuous, and numerical values for the PPE.

III.3. *Rules and the fuzzy indicator of a plural economy*

The inference system follows Mamdani rules,

$$If\,(x_1\,is\,\mathcal{L}_1)\,and\,(x_2\,is\,\mathcal{L}_2)\,then\,\theta_1$$
$$If\,(x_1\,is\,\mathcal{L}_3)\,and\,(x_2\,is\,\mathcal{L}_4)\,then\,\theta_2$$
$$\vdots$$
$$If\,(x_1\,is\,\mathcal{L}_{m-1})\,and\,(x_2\,is\,\mathcal{L}_m)\,then\,\theta_m$$

where $\mathcal{L}_i$, $i=1,..,m$, are linguistic bounds defined in the context of the previous fuzzy membership functions and $\Theta=\{\theta_1,...,\theta_m\}$ is the expected output.

The **plural economy indicator** is the output space $\Theta \in [0,1]$ of the fuzzy inference system. A single value $\theta_i \in \Theta$ of the indicator is calculated after introducing a vector ξ with values of the economic indicators x_1, ..., x_n:

An output value of $\theta_i = 1$ indicates a full compliance with the principles of a plural economy, and a value of $\theta_i = 0$ means non-compliance with these principles. Because fuzzy sets are continuous in [0, 1], values among (0, 1) will indicate a different *degree* of compliance with the PPE: a value of θ_i close to one, indicates a high degree of compliance with the PPE (in general as $\theta_i \rightarrow 1$), while values of θ_i close to zero indicate relative non-compliance with these principles (in general as $\theta_i \rightarrow 0$).

IV.   Results

This section calibrates the membership functions and the rules of the inference system that allow to calculate the plural economy indicator.

It is wise to state at the outset that a proper indicator of plurality should consider both the constitutional PPE and the different forms of economic organization in a plural economy. Nevertheless, in order to keep a simple, clear, illustrative, and understandable explanation of the model, only two of the ten constitutional PPE are considered: redistribution and sustainability. (See also the discussion about the dimensionality of the indicator in the introduction of this paper.)

The study relates the principle of *redistribution* to income redistribution because according to the new State Constitution the development in Bolivia will be assured through an equitable redistribution of economic surplus. The principle of *sustainability*, on the other hand, is related to the concept of environmental sustainability .

IV.1. *Calibration of the input membership functions*

The Gini coefficient was used to measure the principle of redistribution. The Environmental Sustainability Index (henceforth, ES) is used as the indicator of sustainability.

The Gini coefficient is an economic indicator of income redistribution. This index is the most commonly used measure of inequality and takes values between 0 and 100, with zero interpreted as no inequality (Litchfield, 1999). On the other hand, ES benchmarks the ability of nations to protect the environment over the next several decades, by integrating data sets that track natural resource endowments, past and present pollution levels, environmental management efforts, and the capacity of a society to improve its environmental performance. The range of the ES indicator is also between [0; 100], but in this case the higher a country's ES index, the better positioned it is to maintain favorable environmental conditions into the future. See Esty et al. (2005) for details.

Figure 1 shows the histogram of the Gini index based on the data from the World Bank for all the countries in the world between the years 2005-2006. This figure also displays the histogram of the ES index for the year 2005, based on data of 146 countries around the world.

The world's average Gini index in the year 2005 was 42,96 (with a standard deviation of 8,76), and the world's average sustainability index in the same year was 49,88 (with a standard deviation of 8,48). These historical values of ψ_r = 42,96 and σ_r = 8,76 were used to calibrate the parameters of the Gaussian membership function $g\ (x;\ \sigma_r,\ \psi_r)$ which models middling income redistribution (Table 1 and Figure 2). The values ψ_s = 49,8 and σ_s = 8,48 of the ES index were used to calibrate the membership function of middling environmental sustainability (Table 2 and Figure 2).

The function $z\ (x;\ \zeta_{r1},\ \zeta_{r2})$ with $\zeta_{r1} = 0$ and $\zeta_{r2} = 100$ was used to model the equality in the redistribution of income, and the function $s\ (x;\ \omega_{r1},\ \omega_{r2})$ with $\omega_{r1} = 0$; and $\omega_{r2} = 100$ was used to model the inequality in the income redistribution. Being $z\ (\cdot)$ a decreasing function, as $z\ (\cdot) \to 0$, it signals an ideal situation of equity in the income redistribution. Since $s\ (\cdot)$ is an increasing function, as $s\ (\cdot) \to 100$, it characterizes more inequality in the redistribution of income (Figure 2).

The input membership functions of the sustainability index were also an *s*-function $s(x; \omega_{s1}; \omega_{s2})$ and a *z*-function $z(x; \zeta_{s1}, \zeta_{s2})$ with parameters $\omega_{s1} = 0, \omega_{s2} = 100, \zeta_{s1} = 0, \zeta_{s2} = 100$. In this case the *s*-function models the environmental sustainability improvement and the *z*-function models the worsening of the index.

Table 1: CALIBRATION OF MEMBERSHIP FUNCTIONS: REDISTRIBUTION

	Function	Parameters	Calibration
Equal redistribution	z-function $z(x; \zeta_{r1}, \zeta_{r2})$	ζ_{r1} ζ_{r2}	0 100
Middling equal	Gaussian $g(x; \sigma_r, \psi_r)$	σ_r ψ_r	8,76 42,96
Unequal redistribution	s-function $s(x; \omega_{r1}, \omega_{r2})$	ω_{r1} ω_{r2}	0 100

Table 2: CALIBRATION OF MEMBERSHIP FUNCTIONS: SUSTAINABILITY

	Function	Parameters	Calibration
Unsustainable	z-function $z(x; \zeta_{s1}, \zeta_{s2})$	ζ_{s1} ζ_{s2}	0 100
Middling sustainable	gaussian $g(x; \sigma_s, \psi_s)$	σ_s ψ_s	8,48 49,88
Sustainable	s-function $s(x; \omega_{s1}, \omega_{s2})$	ω_{s1} ω_{s2}	0 100

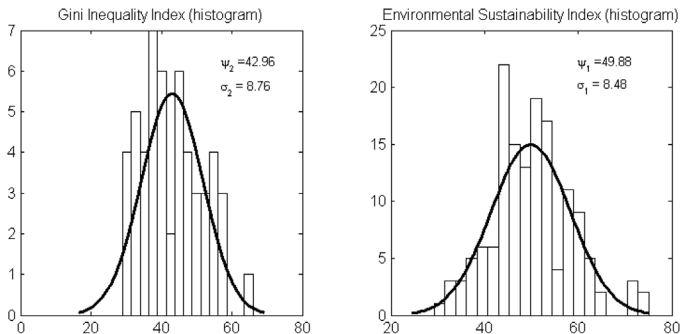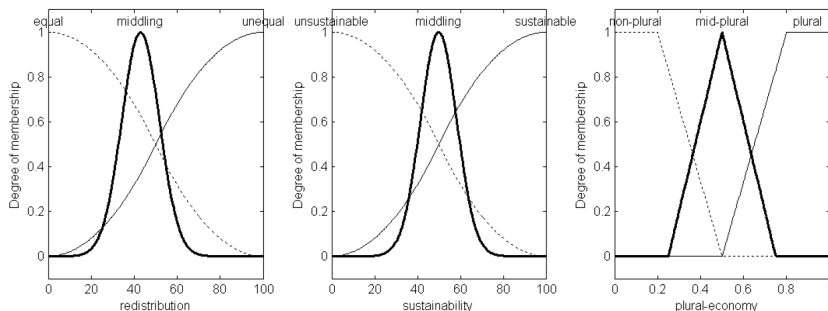### Figure 1: ECONOMIC INDICATORS OF REDISTRIBUTION AND SUSTAINABILITY

Figure 2: MEMBERSHIP FUNCTIONS

IV.2. *Mamdani rules, fuzzy inference system and output membership function*

Let x_1 denote the Gini index and x_2 the ES indicator. The Mamdani rules of the plural economy inference system were:

- IF (x_1 indicates *unequal redistribution*) AND (x_2 indicates *unsustainability*) THEN (Θ indicates *a non-plural economy*);

- IF (x_1 indicates *middling redistribution*) AND (x_2 indicates *sustainability*) THEN (Θ indicates *a middling plural economy*);

- IF (x_1 indicates *middling redistribution*) AND (x_2 indicates *middling sustainability*) THEN (Θ indicates *a middling plural economy*);

- IF (x_1 indicates *an unequal redistribution*) AND (x_2 indicates *middling sustainability*) THEN (Θ indicates *a middling plural economy*);

- IF (x_1 indicates *an equal redistribution*) AND (x_2 indicates *sustainability*) THEN (Θ indicates *a plural economy*),

being Θ the plural economy indicator. Fuzzy sets were used to translate the linguistic rules into a quantitative economic indicator, using trapezoidal and triangular output functions to measure the degree of compliance with the PPE.

The values of $\delta_{11} = 0, \delta_{12} = 0, \delta_{13} = 0,20, \delta_{14} = 0,50$ of the output function *trap* $(x; \delta_{11}, \delta_{12}, \delta_{13}, \delta_{14})$ depict a situation of a non-plural economy between 0 and 0,20, and as *trap* $(\cdot)$ linearly approaches 0,50 reaches a middling plural economy. The trapezoidal function *trap* $(x; \delta_{21}, \delta_{22}, \delta_{23}, \delta_{24})$ with $\delta_{21} = 0,50, \delta_{22} = 0,80, \delta_{23} = 1, \delta_{24} = 1$ starts at a middle state of plural economy and piecewise linearly approaches 1 with progressive enhancements. The triangular function *tri* $(x; \tau_1, \tau_2, \tau_3)$ with $\tau_1 = 0,25, \tau_2 = 0,50; \tau_3 = 0,75$ models the middle state of the indicator ranging between a complete plural economy and a non-plural economy (Table 3, Figure 2).

The complete inference system used to calculate the plural economy indicator can be appreciated in Figure 3. The diagram shows that if a vector ξ with values of the indicators of redistribution and sustainability were introduced into the membership functions, the inference system would output a scalar θ_i with a value of the plural economy indicator.

Figure 3: INFERENCE SYSTEM
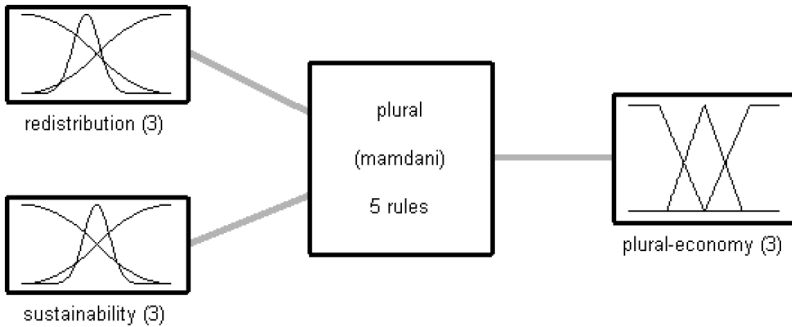
**Table 3: CALIBRATION OF MEMBERSHIP FUNCTIONS:
PLURAL ECONOMY**

	Function	Parameters	Calibration
Non-plural economy	trapezoidal $trap\,(x;\delta_{11},\delta_{12},\delta_{13},\delta_{14})$	δ_{11}	0
		δ_{12}	0
		δ_{13}	$0,20$
		δ_{14}	$0,50$
Middling plural	triangular $tri\,(x;\tau_1,\tau_2,\tau_3)$	τ_1	$0,25$
		τ_2	$0,50$
		τ_3	$0,75$
Plural economy	trapezoidal $trap\,(x;\delta_{21},\delta_{22},\delta_{23},\delta_{24})$	δ_{21}	$0,50$
		δ_{22}	$0,80$
		δ_{23}	1
		δ_{24}	1

IV.3. The plural economy indicator

Figure 4 shows a surface view of the plural economy indicator Θ, as a function of the redistribution and sustainability indicators previously discussed. Also, a bivariate projection of the surface is depicted on Figure 5.

As it can be seen, the plural economy indicator Θ is a non-linear decreasing function of the redistribution indicator, i.e. Θ indicates a high degree of plurality when the redistribution is close to 0 (expressing total equality) and the indicator falls to zero as the value of redistribution approaches a value of 100 (maximal inequality).

At the same time, the plurality indicator Θ is an increasing nonlinear function of the indicator of sustainability. If the indicator of sustainability increases (signaling more environmental sustainability) then Θ increases too, being sustainability an ideal principle of a plural economy.

IV.4. Numerical examples of the plural economy indicator

As examples of using the plural economy indicator Θ, vectors with real and hypothetical values of the Gini index and the ES index will be used in the inference system for a plural economy.

In the year 2005, before the socialist movement led the government in Bolivia, the Gini index of Bolivia was equal to 58, and the ES indicator of sustainability was equal to 59,5. Using these values, a vector ξ_1 = [58 59,5] was introduced into the inference system, and the output value of the plural economy indicator was equal to θ_1 = 0,5065 (Table 4). The indicator suggests, as expected, a poor compliance with the principles of a plural economy in the year 2005.

Figure 4: SURFACE-VIEW OF THE PLURAL ECONOMY INDICATOR (Θ)
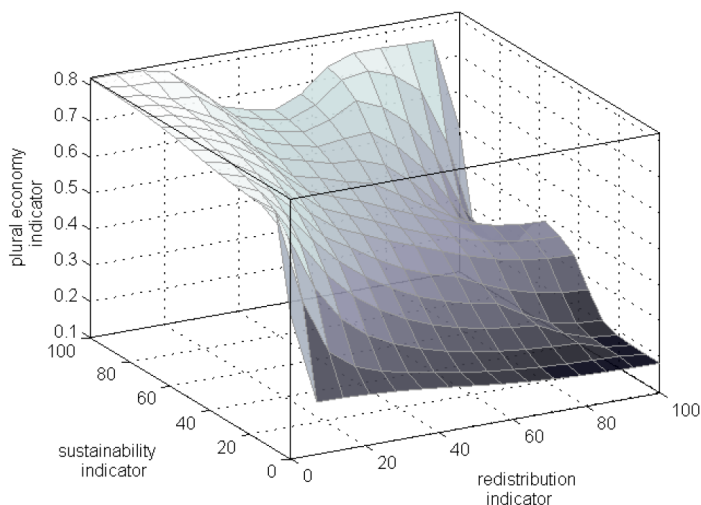
No information of the ES index is available after the year 2005, and the most recent information of the Gini Index in Bolivia dates the year 2007. In this year, the Gini index was equal to 57. Assuming no change in the ES index between these years, a second vector ξ_2 = [57 59,5] yields an indicator of plurality equal θ_2 = 0,5113, indicating a negligible improvement of 0,0048 in the compliance with the principles of a plural economy. This last result, nevertheless, should be taken with caution, as it is based on the assumption of no improvement of the environmental sustainability indicator during the years 2005 to 2007.

The availability of updated data certainly constrains a current assessment of compliance with the principles of a plural economy.

However, a simulation exercise can be useful to prove the ability of the indicator Θ to properly measure the progress towards a plural economy.

Presume a situation in which the Bolivian government improves the income redistribution and enhances the environmental sustainability to a Gini index of 35 and an ES index of 85, respectively. Then, if a vector with these values were introduced into the inference system, the plural economy indicator would be equal to 0,6653. This value would certainly support the claim that Bolivia is becoming a plural economy.

Table 4: PLURAL ECONOMY INDICATOR Θ

Input vector $\boldsymbol{\xi}^{\dagger}$	Output scalar θ_i
$\boldsymbol{\xi}_1 = \begin{bmatrix} 58 & 59,5 \end{bmatrix}$	0,5065
$\boldsymbol{\xi}_2 = \begin{bmatrix} 57 & 59,5 \end{bmatrix}$	0,5113

† The value of $\xi_{11} = 58$ is the Gini index of Bolivia for the year 2005 (source: TheWorld Bank) and the value of $\xi_{12} = 59,5$ is the value of the Environmental Sustainability Index for Bolivia for the year 2005 (source: Yale Center for Environmental Law and Policy, Yale University, and Center for International Earth Science Information Network, Columbia University). No information is available for the ES after the year 2005, and the value of the Gini index in the year 2007 for Bolivia was used in $\xi_{21} = 57$

Figure 5: BIVARIATE PROJECTIONS
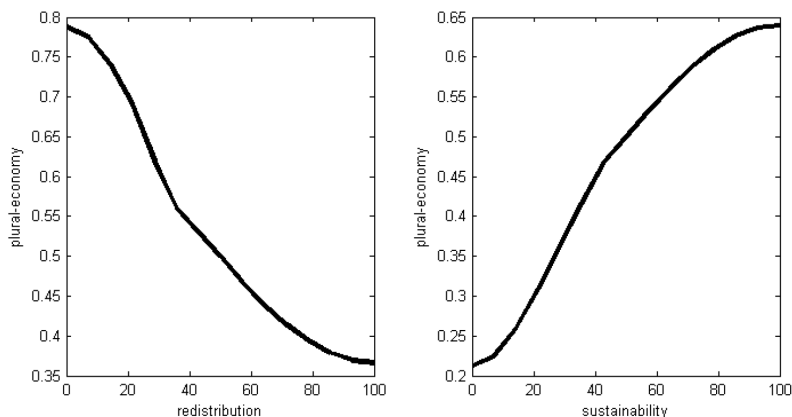
V.   Conclusion

The aim of this paper was to propose a plural economy indicator based on fuzzy sets. A complete inference system was designed and membership functions were calibrated to calculate the indicator with real and hypothetical data of income redistribution and environmental sustainability.

Although only two of the ten constitutional PPE were considered, the empirical and simulated outcomes showed that the indicator translates the redistribution and sustainability indicators to an overall measure of the plurality of the Bolivian economy. Further research is needed to extend the model with the remaining constitutional principles and with measures of the different forms of economic organization. (For example, the *World Bank's Strength of Legal Rights Index* could be used to account for the principle of *legal certainty*, and the *Transparency International's Corruption Perception Index* may be a suitable approximation for the principle of *transparency*.)

Based on the available data of redistribution and sustainability, the results of the plural economy indicator suggested a low compliance with the PPE in Bolivia in the year 2005. The lack of updated data for the year 2011 constrained the possibility to evaluate whether the socialist government has actually attained improvements towards a more pluralistic economy in Bolivia.[6] However, if new data of redistribution and sustainability became available, the indicator proposed in this study can be used to measure quantitatively the compliance with the principles of a plural economy, and thus objectively judge the effect of the economic changes in Bolivia.

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6   Two policies of income redistribution of the Bolivian government were (*i*) the conditional cash transfers *Renta Dignidad, Juancito Pinto, Juana Azurduy*, and (*ii*) the access to productive capital through the second-tier bank BDP (*Banco de Desarrollo Productivo*). If these policies make the income redistribution more equitable, this should be reflected as improvements in the plural economy indicator.

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